The conservation of momentum which is required for you to know it, how the fluid particles are moving it, what could be the force exerting on that, what could be the velocity. Similar way, we can understand the energy conservation which plays a major role for us when the fluid comes from one location to other locations, how much of work is done by the fluid or into the fluid.

Similar way, whether there is heat transfers happening which you can feel it, if there is a temperature gradient there will be heat transfer either to the surrendering of the systems or into the systems or out of the system, that is what we can do. So, to summarise this, that means, we all know that there are three energy conservation principles that we follow in solid mechanics when you consider as a system.

Same concept also we can use at the system levels to solve the problems, conservation of mass, conservation of linear momentum which is Newton's law, and the conservation of energy which is the first law of thermodynamics. As I discussed, there is a system and control volume. Let us understand the Reynolds transport theorem which establish the relationship between the conservation law at the system level and the conservation at the control volume level.

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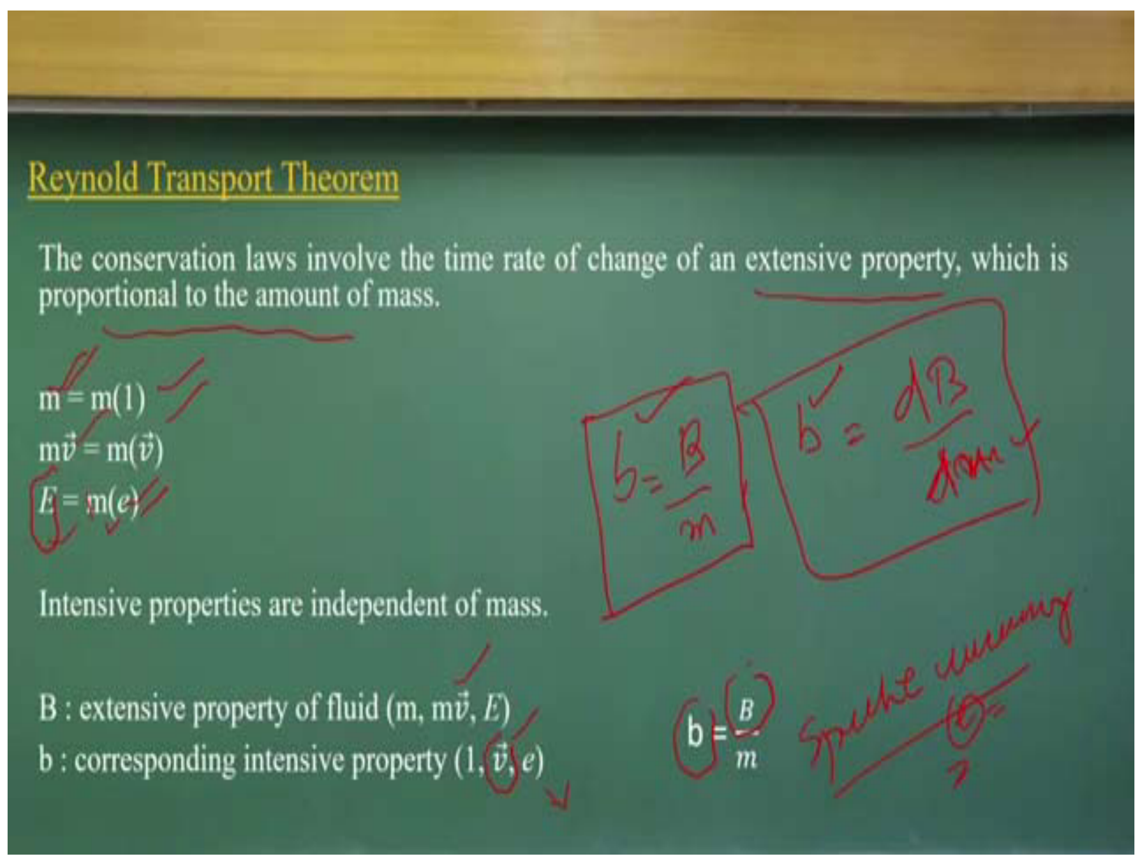

Now, let me define two types of properties that we have; one is called extensive property and the other is the intensive property. The extensive property which is considered as proportional to the amount of mass. When you apply extensive properties, that means you are the properties which are proportional to the amount of mass. That means, as mass increases you will have extensive properties going to increase.

Mass decreases, the extensive property decreases. It is proportional to the mass. For example, as we discussed it in three basic laws, we talked about mass conservation, momentum conservation, and energy conservation. So, m will be the mass conservation part, momentum and energy conservation. But when you look at the intensive properties, that means it is independent of mass, that means, which is denoted as

$$b = \frac{B}{m} = \frac{dB}{dm}$$

So, if you look it that way, there are two properties, extensive property and intensive property. In intensive property independent to mass or $\frac{B}{m}$, per unit mass what we are talking about. For example,

$$m = m(1)$$
$$m\vec{v} = m(\vec{v})$$
$$E = m(e)$$

for energy conservation the extensive property will be the one, but in the case of the momentum, but intensive property will be the velocity vectors. Similar way, if you look for energy conservation, if you look at extensive properties, that is energy.

But intensive property is e which is the specific energy. That means energy per unit mass. So, e is independent to the amount of mass in the control volume or system level. So, we define the difference between extensive property and intensive property. Extensive property we define as B, intensive property we define as b. They have the relationship, simple relation like this, mathematically dB by dm. That is the relationship that is there.

$$b = \frac{B}{m} = \frac{dB}{dm}$$

And we define the difference between the extensive properties in three cases, mass, momentum, and energy, but correspondingly for intensive properties which is independent of mass will be one velocity vectors and e stands for specific energy. Now, we will go to derive Reynolds transport theorem. The derivation of the Reynolds transport theorem are available in almost all the fluid mechanics books.

The idea for me is to introduce the Reynolds transport theorem so that you can easily understand it. But the step wise derivations, if you are not understanding it, I could suggest you to follow any of the fluid mechanics books, F.M. White, Cengel Cimbala, or any other advanced fluid mechanics books, you can see the derivations of Reynolds transport theorem. Only the symbol of representation of extensive properties, intensive properties, either B or b or β are used in different books in different forms.

Otherwise, the Reynolds transport theorem which is the basic equations, the derivation of this equations is available almost in all fluid mechanics books.

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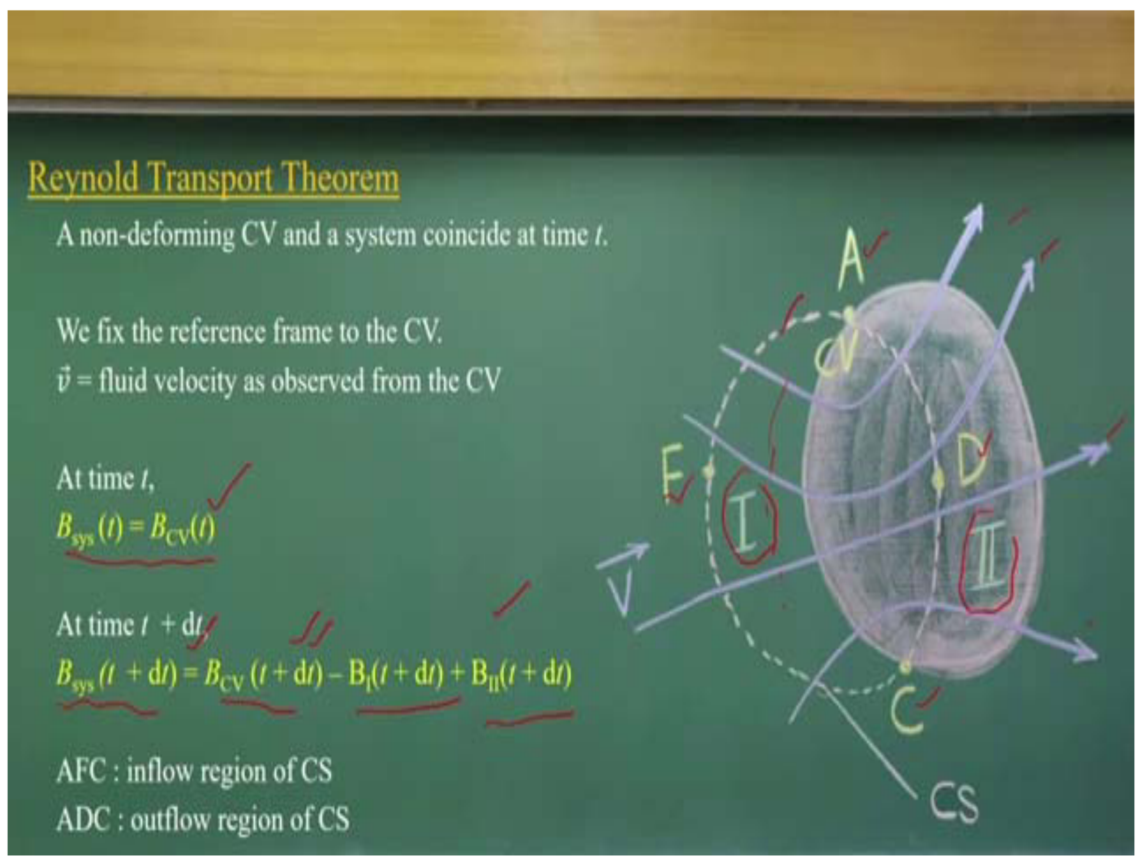

Now, let us come to the derivations which I will highlight as I say it while derivation, which are the major components, not line by line. So, first, what we are considering is a non-deforming control volume. And this is my non-deformable control volume and also I have drawn the streamlines representing the flow that is coming in and coming out. So, if this is my control volume, I can define there will be a control surface defined by A, D, C, and F.

$\vec{v}$ = fluid velocity as observed from the CV

This is the gas part that is the control surface and this part is indicating, these lines or the streamlines are indicating how the influx is coming into the control volume and going out from the control volume, through this surface. That is what is my control volume. That means, whatever the fluid particles present at time t, that will represent the control volume and the reason is these are fixed. Let me have a very simplified case.

I will consider that is the system for me at time t. That means I consider at time t whatever the fluid particles are there within this control volume that is what the system is. As already I illustrated, at $t + dt$, at the next instant of the time, definitely the fluid particles there will move, as we can see from the velocity vectors, they will move out from this and they can occupy the space, okay. This can occupy this space.

At time t,

$B_{sys}(t) = B_{CV}(t)$

At time $t + dt$,

$B_{sys}(t + dt) = B_{CV}(t + dt) - B_I(t + dt) + B_{II}(t + dt)$

Now, I can define this 3 into 3 different spaces, like the space defined by this part I can give it as I and this can be II and this can be used as the control volume space, the space is occupied at the system at t time as well as system at $t + dt$ time. So, I have defined these regions into three parts, one is the influx region, the other is outflux region, another is the common region which is there when the system at t time and also t plus delta time.

So, there is influx region, there is outflux regions. So, at the system level $t + dt$ I can define it at control volume level of $t + dt$. The positive and negative you can understand. We are defining in terms of in or out. That is the sign convention that you can try to understand when I talk about the velocity and area dot products, that is influx and outflux will have different signs, that is what we will discuss.

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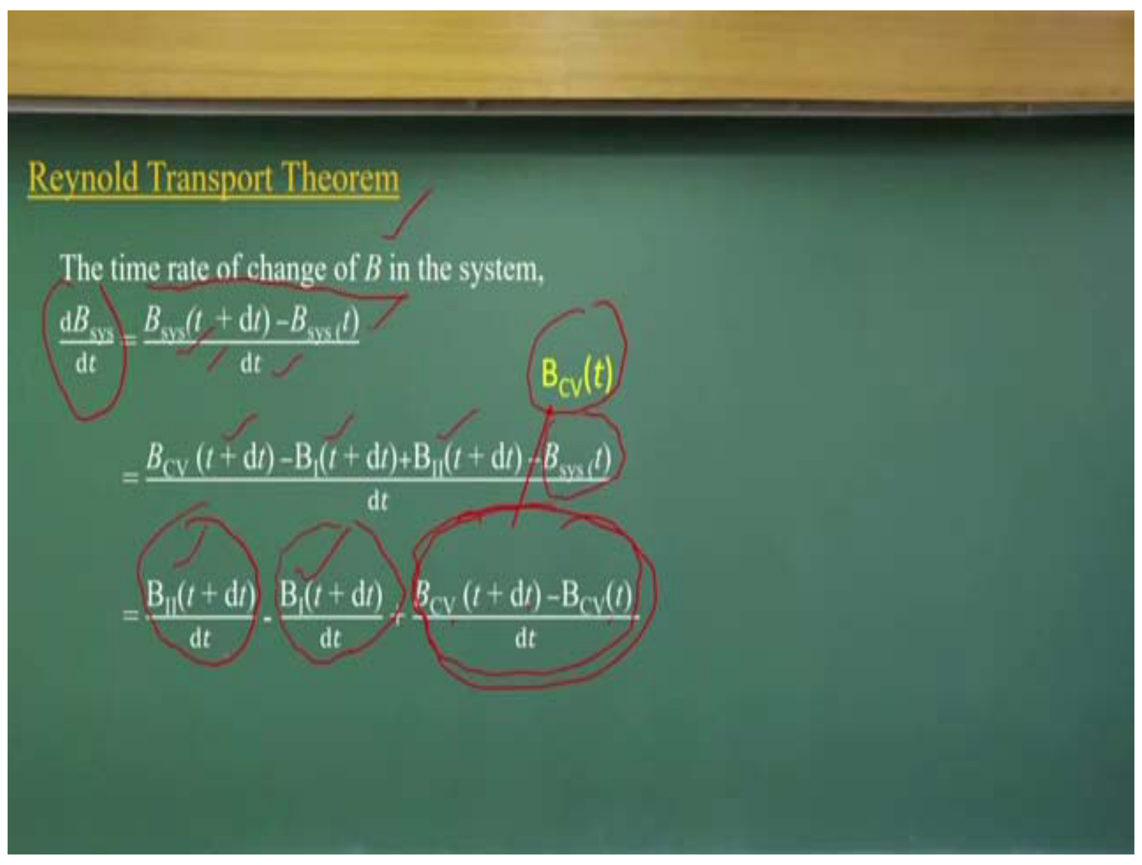

Now, if I look at simple definitions, the calculus is that the time rate of change of B in the system as a definition $t + dt$ minus the system at the t level, and I just apply what I have derived, the three components at the three regions, replacing these values. Again, I know, the B value of extensive property at t is equal to the B value of extensive property of control volume at t. The time rate of change of B in the system,

$$\frac{dB_{sys}}{dt} = \frac{B_{sys}(t + dt) - B_{sys}(t)}{dt}$$

So, I replace this value at the system to the control volume level. If I just do a rearrangement of this equation, I will get one part, you can understand it, at the control volume level which is showing B control volume at $t + dt$ time, B control volume at t time by dt. That means, what is the time rate of change happening at the control volume level, that is the definition what we will get.

$$\frac{dB_{sys}}{dt} = \frac{B_{II}(t + dt)}{dt} + \frac{B_I(t + dt)}{dt} + \frac{B_{CV}(t + dt) - B_{CV}(t)}{dt}$$

And we have other two parts which is representing influx and outflux of these regions, of region 2 and 1, $t + dt$ by dt and $t + dt$ by dt for B_I and B_{II} which are the different regions, the influx and the ouflux regions. So, you can understand it. We get dB by dt at the system level can be composed of three parts. Now, I will discuss these three parts to simplify it.

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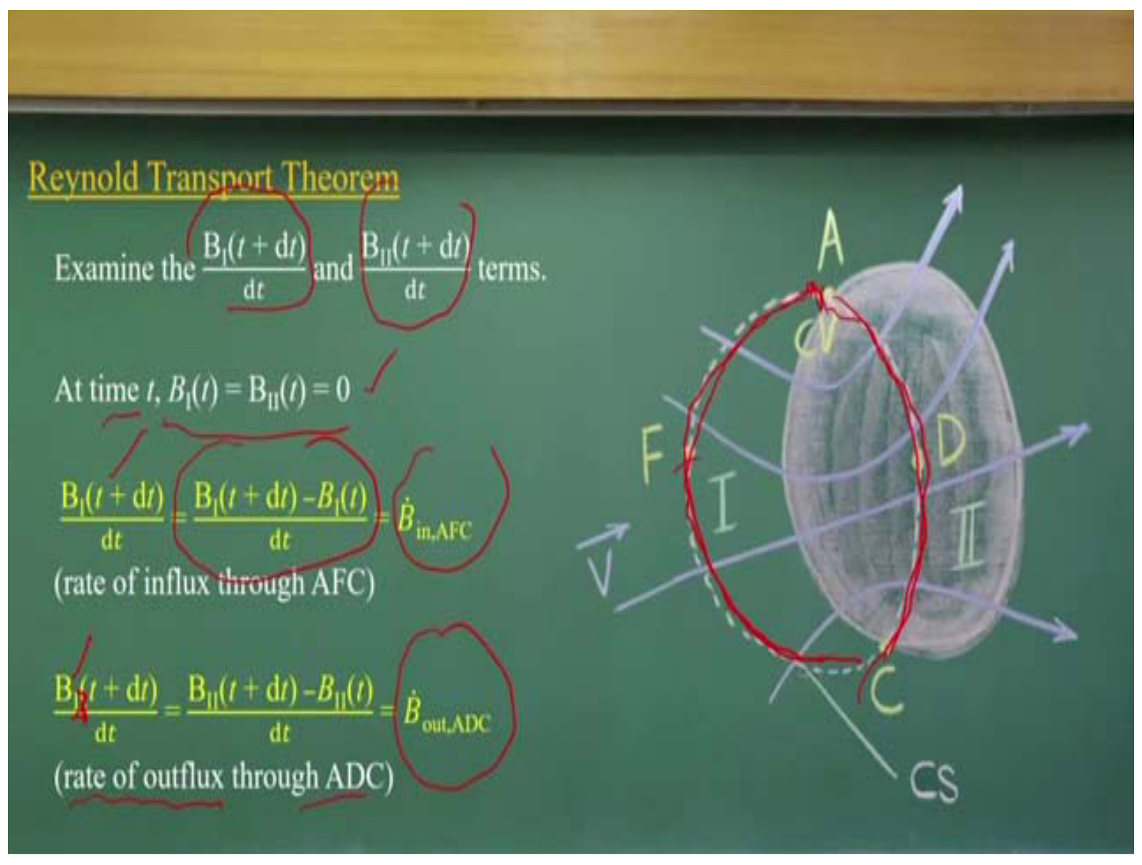

Let us examine this part, okay? Examine the $\frac{B_I(t + dt)}{dt}$ and $\frac{B_{II}(t + dt)}{dt}$ terms

What is the time rate of change of B_I at $t + dt$ time or B_{II} at $t + dt$ time? As you know it, At time t, $B_{I}(t) = B_{II}(t) = 0$

$$\frac{B_{I}(t + dt)}{dt} = \frac{B_{I}(t + dt) - B_{I}(t)}{dt} = \dot{B}_{in,AFC}$$

what is representing this? That is representing rate influx of B through the surface of A, F and C. Similar way, you can find out,

$$\frac{B_{II}(t + dt)}{dt} = \frac{B_{II}(t + dt) - B_{I}(t)}{dt} = \dot{B}_{out,ADC}$$

which will be again the rate of outflux through this A, D, and C. So, the two components we define it, one is representing influx and the other is representing the outflux, how much of rate is happening through a surface of A, F, C or through the forces of A, D, C. That is what we represent.

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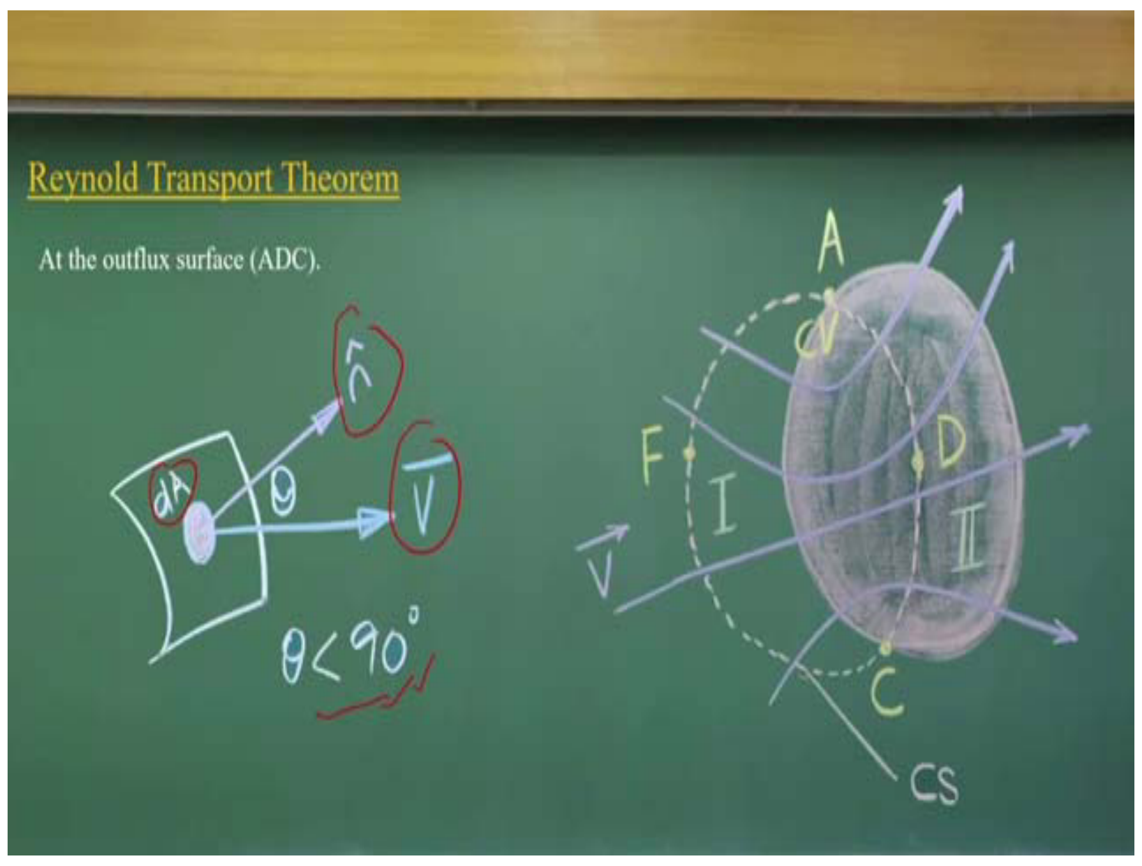

Now, how to compute the outflux surface, okay? That means, assuming it is a three-dimensional control volume, and over that surface I want to integrate it, I want to know how much of influx is coming into the control volume or going out from control volume. That means I can take a small area dA and I can have a normal vector and I can have the velocity vector to that.

So, if theta is less than 90° which is representing the outflux, the flow is going out from this control surface. If I have this condition of dA area and I try to compute what could be the flux going out from this surface during the time dt.

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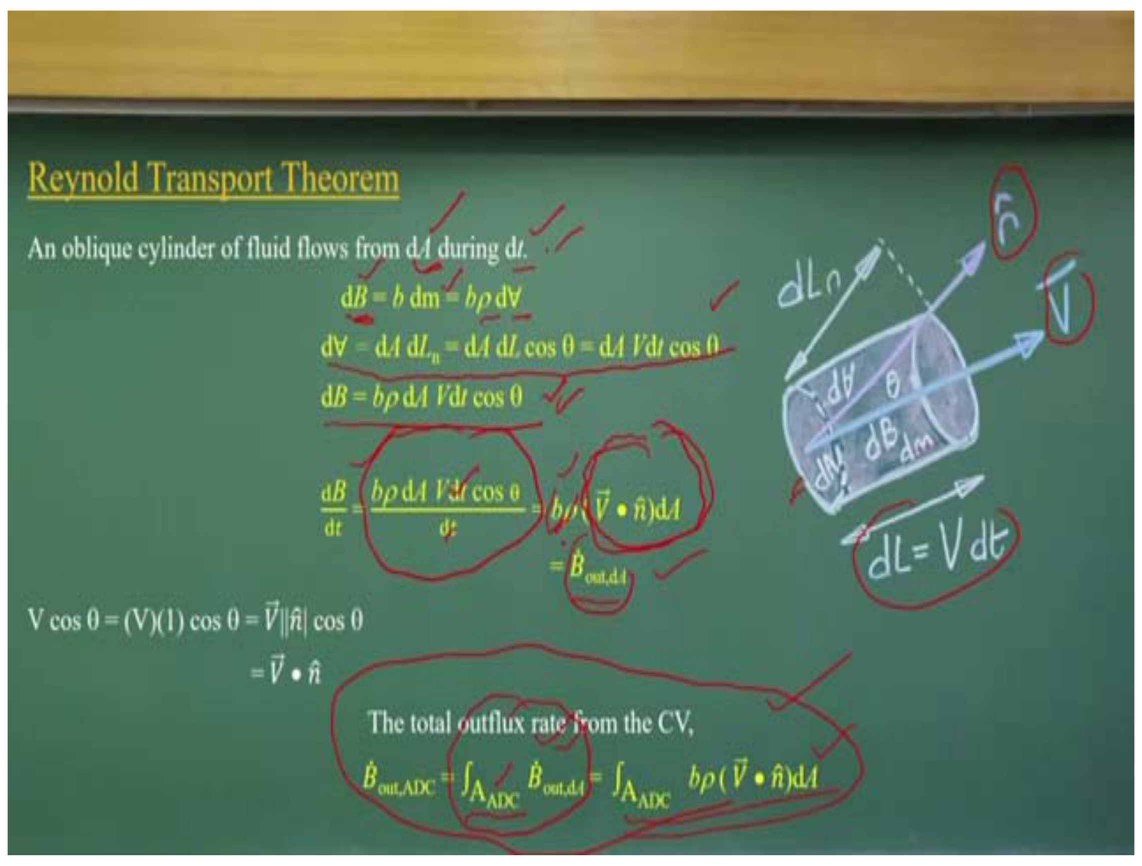

That part if I look, very simple thing, the same element area, I have the velocity vectors, n is the normal unit vector of these things. And if we are in dt time, if you know this velocity V, then the length of this imaginary volume will be velocity and time will be the length, that means what is the volume of the space because of this outflux. That will be V into dt, that is what is the length and we know this area. Now, we have to compute how much is coming through this oblique cylindrical surface which has dA area and during time dt. You can have

$$dB = b\,dm = b\rho\,d\forall$$

dm is representing the elemental mass, the mass will be the density in times of the volume of the surface, and we can use a simple geometry to find out the volume which will be area into length, then we can find out the length projections, then we can convert the length dL equal to V into dt. So, we will get dB will be this part.

$$d\forall = dA\, dL_n = dA\, dL \cos\theta = dA\, V dt \cos\theta$$

$$dB = b\rho\, dA\, V dt \cos\theta$$

If you have time rate of the dB, if you look it, you get this part where dt is cancelled out, okay, and if you look, $V dt \cos\theta$ is nothing else, it is a dot product of the velocity vector and the unit vector into dA which is the outflux representation.

Since I have a big control volume and the surface is irregular, then I can integrate that through a surface of A, D, C to get total outflux rate that is going from this control volume. So, if you can try to look at it with a simple geometry, we can find out how much total outflux rate is going out from this control volume doing a surface integration.

$$\frac{dB}{dt} = \frac{b\rho dA\, V dt \cos\theta}{dt} = b\rho\,(\vec{V} \bullet \hat{n})dA$$
$$= \dot{B}_{out,dA}$$

If I do the integration, then I can find out what is the amount of the total outflux rate that is going out from this control volume, either mass conservation, mass and momentum flux are energy flux. That is what we will discuss more as I proceed.

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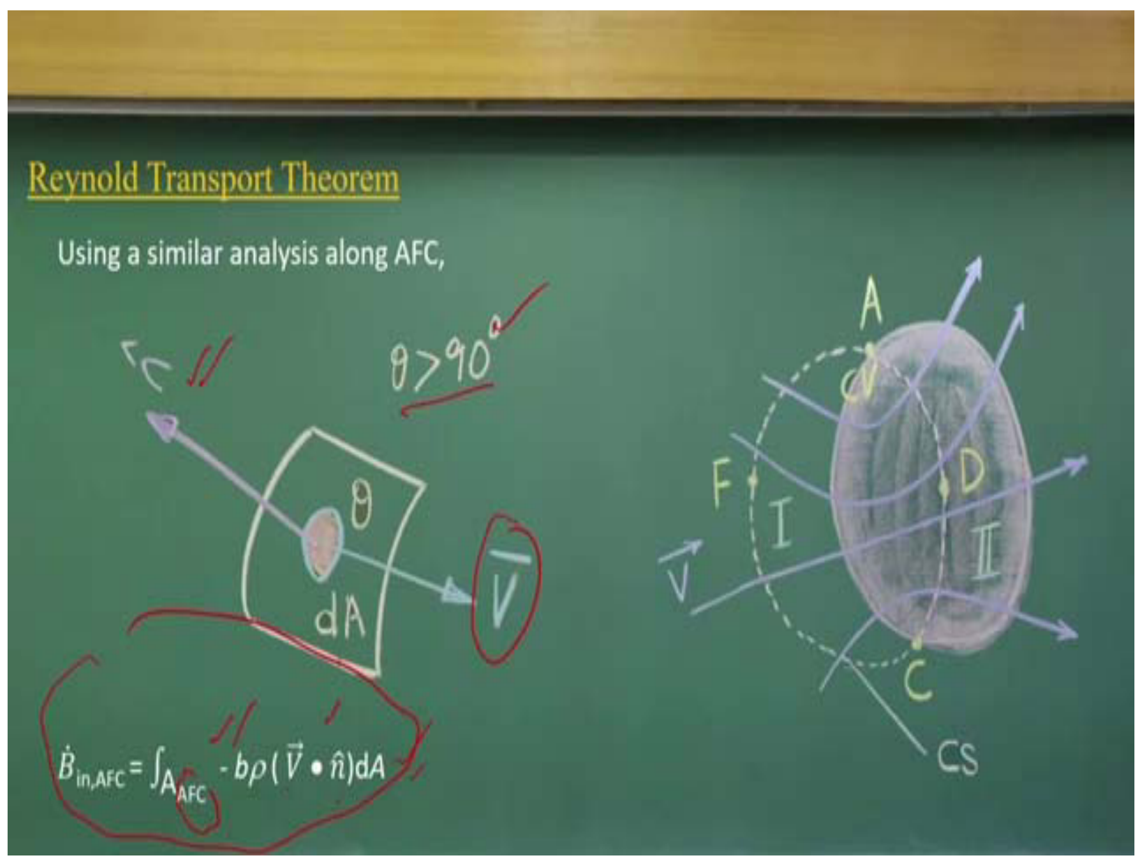

Now, we look at the similar way, if I am to compute how much influx is coming in, so you can understand the velocity will be inward and the surface unit vector will be outwards and your theta will be that. And the same way you can get it, only this sign convention will be different, nothing else, okay. The same way, find out the total influx coming in through this control surface.

It will be a surface integral with respect to AFC, the sign convention indicates which direction it is going on with a dot product of the velocity and the normal vectors of the surface area and then you get the dA part.

$$\dot{B}_{in,AFC} = \int_{A_{AFC}} -b\rho(\vec{V} \bullet \hat{n})dA$$

So, the same derivations as I said it. You try to understand the derivation what I have been talking. If you have any doubt you can follow up any of the fluid mechanics book in the chapter of Reynolds transport theorem.

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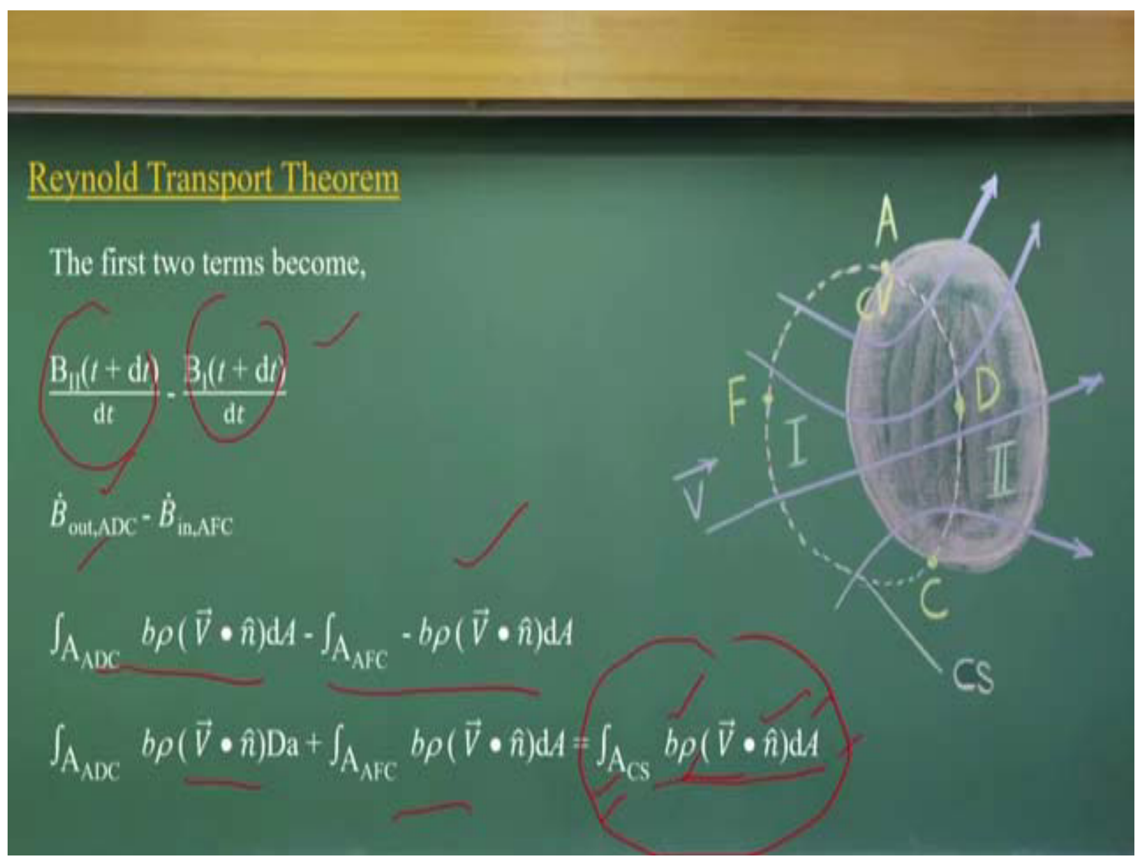

Now, coming to the third part. The first two terms we can write it as earlier you will have the influx and outflux. That part can be written as integrals of influx and outflux and if you combine it next what you are getting? This is total cross section, okay? That means net outflux of mass, momentum, or the energy flux going through the system, either influx or the outflux but as you integrate it it represents the net outflux that is going through this control surface.

The first two terms become,

$$\frac{B_{II}(t+dt)}{dt} - \frac{B_{I}(t+dt)}{dt}$$

$$\dot{B}_{out,ADC} - \dot{B}_{in,AFC}$$

$$\int_{A_{ADC}} b\rho(\vec{V} \bullet \hat{n})dA - \int_{A_{AFC}} -b\rho(\vec{V} \bullet \hat{n})dA$$

$$\int_{A_{ADC}} b\rho(\vec{V} \bullet \hat{n})dA - \int_{A_{AFC}} -b\rho(\vec{V} \bullet \hat{n})dA = \int_{A_{CS}} b\rho(\vec{V} \bullet \hat{n})dA$$

If I know the velocity and the unit vectors, if I know the b and if I know the density, how it varies, then we can find out what could be the net outflux that is coming out if we are doing surface integrals of these functions. It looks easy but it is not that easy, that is what my idea is for you to understand the problems in a better way.

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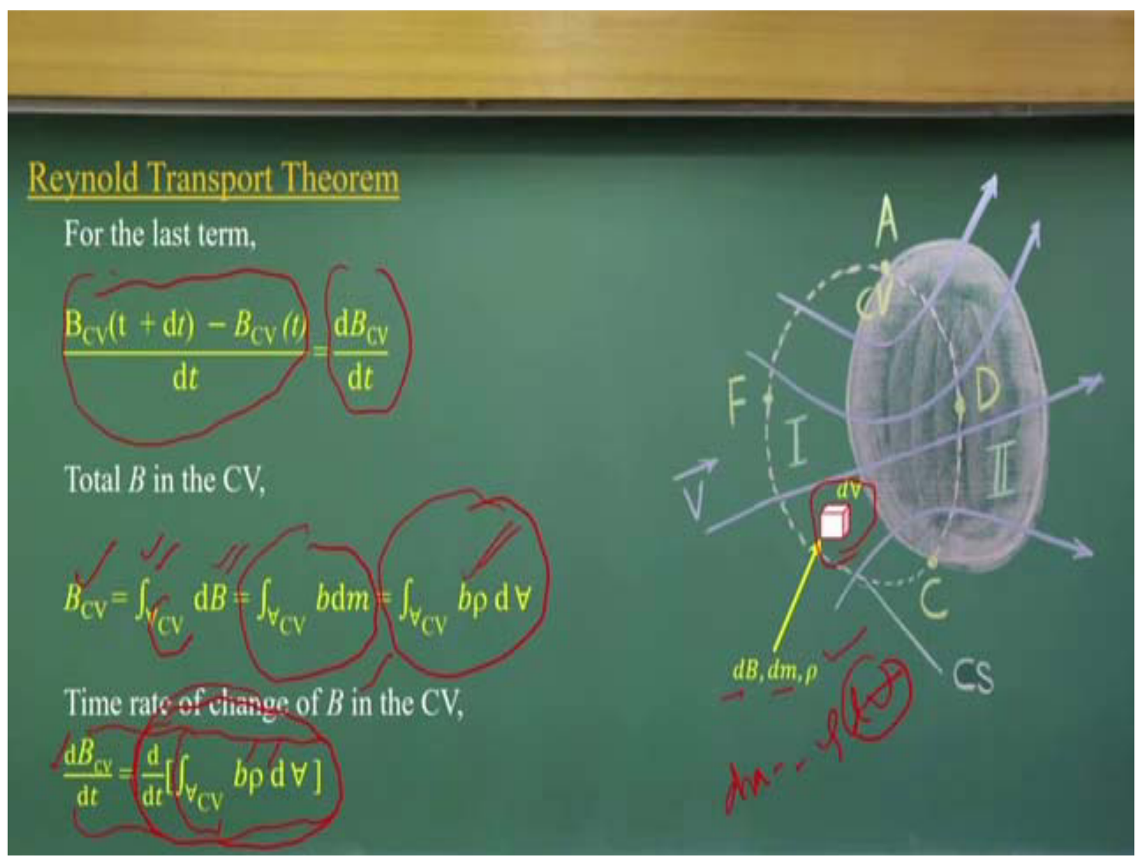

Now, if you look at the last term which is very simplified form, if you look at the definition wise, what is this, the change of the B value

$$\frac{B_{CV}(t + dt) - B_{CV}(t)}{dt} = \frac{dB_{CV}}{dt}$$

How to compute the B_{CV}? What is the total extensive property in the control volume? That means you can do volume integrals of the small control volume what we have considered here.

If ρ is the density, the dm and dv is the mass and this, that means we can just integrate this part and we can have this part.

$B_{CV} = \int_{\forall_{CV}} dB = \int_{\forall_{CV}} b dm = \int_{\forall_{CV}} b\rho \, d\forall$

Very simple definition, density times volume is the mass. If I integrate the volume integrals over this control volume, then I will know what will be the B_{CV}?

Time rate of change of B in the CV,

$$\frac{dBCV}{dt} = \frac{d}{dt}\left[\int_{\forall_{CV}} b\rho \, d\forall\right]$$

That means the time rate change of the control volume mathematically you can write it as this. Just substituting B $_{CV}$ at this point, it is a time rate change of the control volume. This is the volume integral of B density and dv. If I do it, that is what will represent the **(()) (61:35).** Now, I will just use these three terms to form Reynolds transport theorem which we are just discussing.

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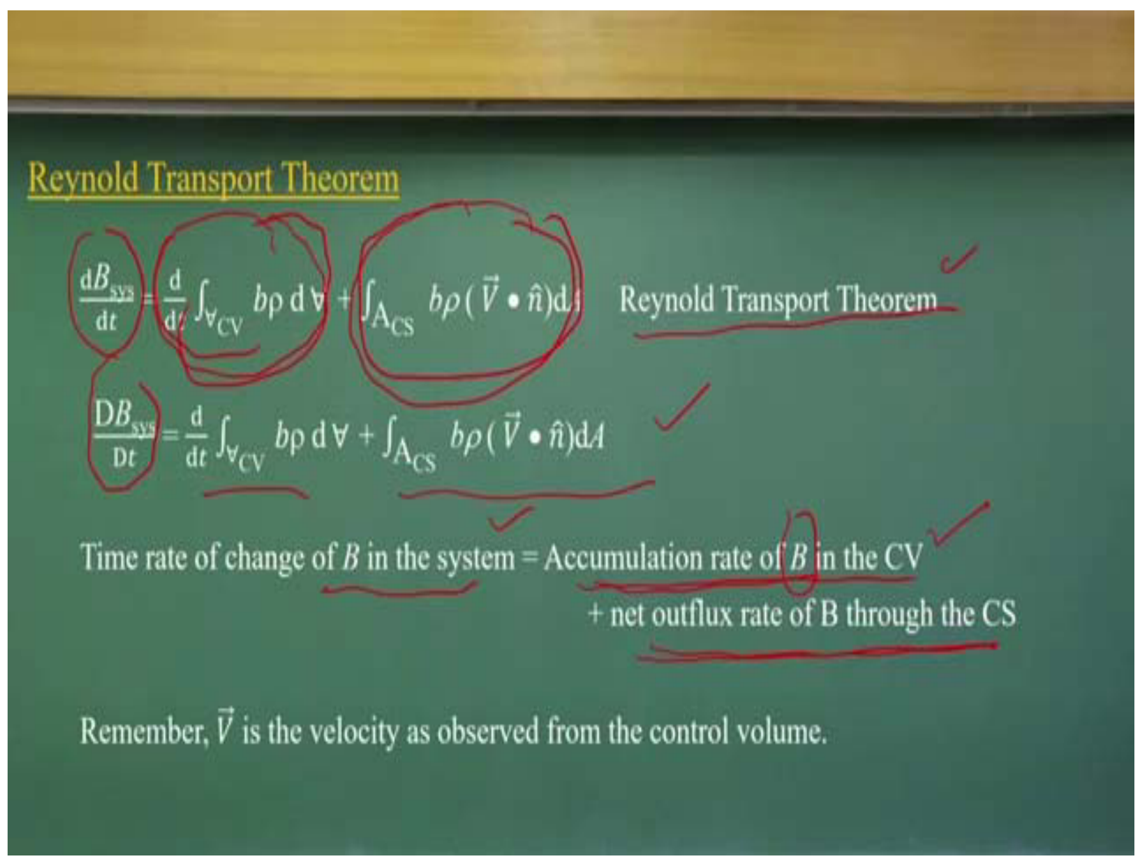

That means the system level of the change of the B value at the time rate of the change of B value at the left side is equal to the, what is the accumulation rate of change of B value at this control volume level which is indicating as volume integrals of B ρ dB. How much change is happening at the control volume levels, how much net outflux of the B through this control surface, that is what is indicating here, okay?

$$\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{\forall_{CV}} b\rho \, d\forall + \int_{A_{CS}} b\rho(\vec{V} \bullet \hat{n})dA \quad Reynold\ Transport\ Theorem$$

And sometimes we use total derivative to represent that one. So, if you look at that, dB system by dt we can have this system to define it. As summary to that I can say that time rate change of the B in the system is equal to, at the control volume level, accumulation rate of the B value in the control volume, net outflux of the B through the control surface. So, remember the velocity that you observed in the control volume level.

$$\frac{DB_{sys}}{Dt} = \frac{d}{dt}\int_{\forall_{CV}} b\rho \, d\forall + \int_{A_{CS}} b\rho(\vec{V} \bullet \hat{n})dA$$

I will talk about how you use the relative velocity component when you go for control volume moving with velocity V. Thus, we look at very complex problem.

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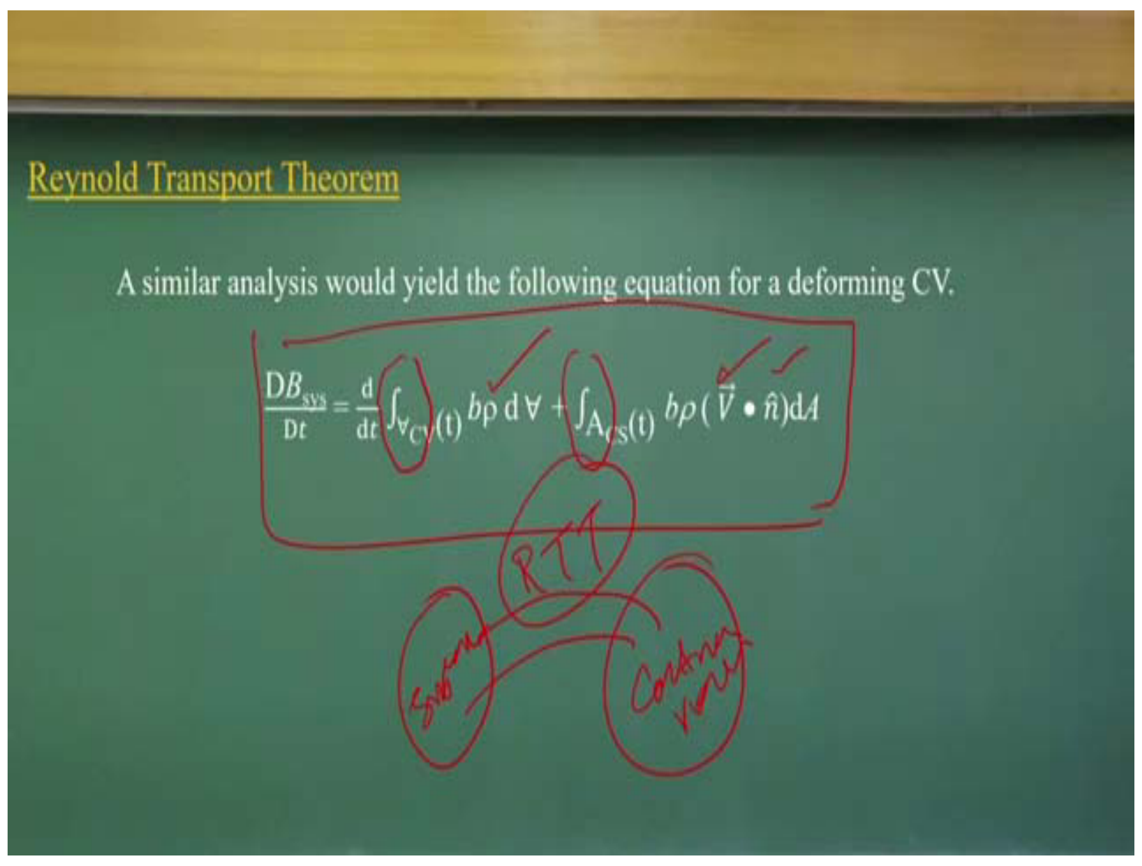

$$\frac{DB_{sys}}{Dt} = \frac{d}{dt}\int_{\forall_{CV}} b\rho\, d\forall + \int_{A_{CS}} b\rho(\vec{V} \bullet \hat{n})dA$$

So, this is what is Reynolds transport theorem, and the basic physics you can understand. It is that we are relating with a system level and the control volume levels. That means Reynolds transport theorem now, if you have a system, you have a control volume, and very simple representation is the relationship is developed by Reynolds transport theorem which looks like mathematically very complex problem.

Now, we have volume integrals, we have surface integrals, we do not know how density varies it, how the velocity varies it and how it is related with the system to control volume level. But this is what is my duty, to simplify this complex equation and solve many problems. That is what I will do in the coming five to six lectures.

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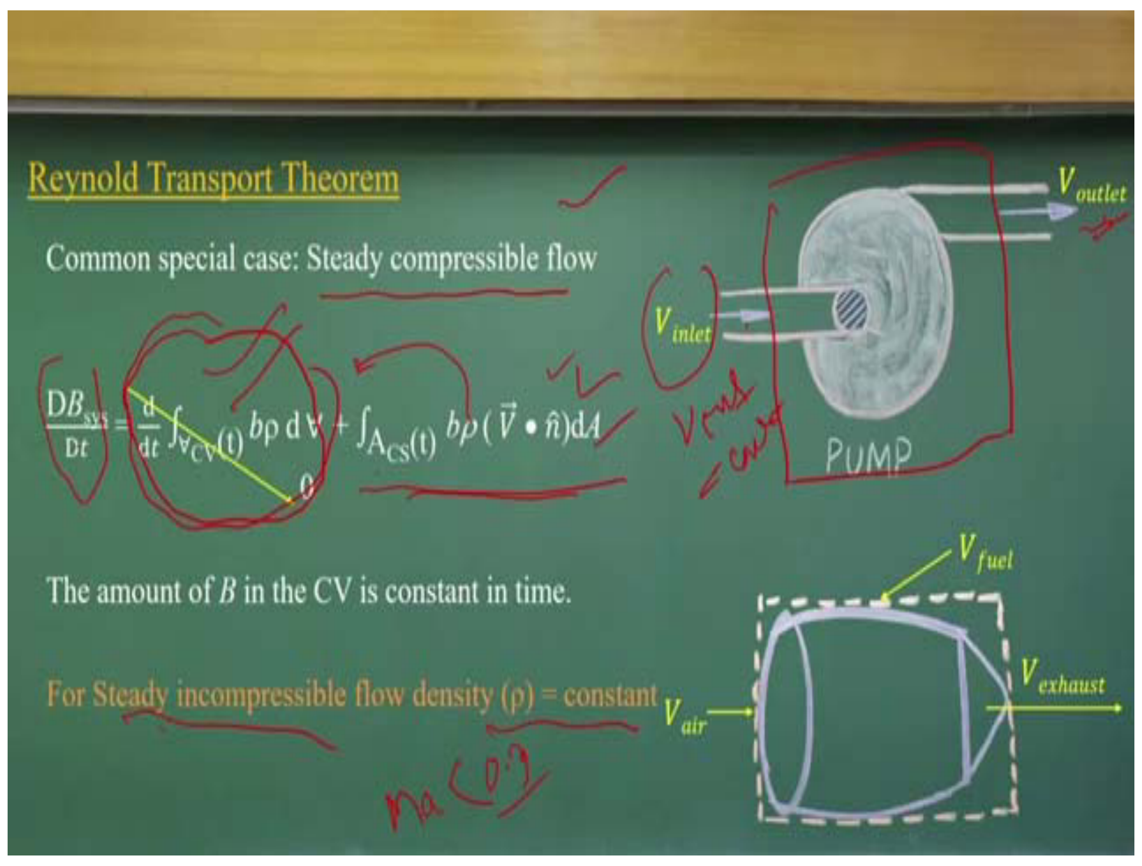

Let us come back to very simple case that we can do, the steady incompressible flow, okay. If you consider a steady incompressible flow, that means the flow does not change with time, the density changes in this case of the steady compressible flow.

$$\frac{\mathrm{D}B_{\mathrm{sys}}}{\mathrm{D}t} = \underbrace{\frac{d}{dt}\int_{\forall_{CV}} b\rho \, d\forall}_{0} + \int_{A_{CS}} b\rho(\vec{V} \bullet \hat{n})dA$$

If that is the condition, your major part of this becomes 0, because if d by dt becomes 0. Let us consider this case. I have V inlet coming and V outlet is going out and I have a pumping system.

If I consider the V inlet is constant, you can observe it, your V outlet will be constant, steady state will come, it you make it, the inflow is constant. After a certain time if you look, the V out is constant. So, there will be no change. There will be flow but there is no change in the velocity influx and outflux, then what it indicates is that there is no net change in storage or accumulated mass momentum flux within the control volume.

That becomes 0. If it is that, then, dB by dt system is equal to this part. Just we are going to do surface integral to solve how the system is changing it at the control volume level. So, that is the simplification if you use steady compressible flow. But most of the time as you use the steady incompressible flow density is constant, the mac number what you consider the flow is less than 0.3, then your density will come out. So, the problem becomes more simple.

You just do integration of velocity and the unit vector product with the B value which could be specific energy, could be the velocity vectors, or could be 1. So, the problem becomes simpler when you consider steady incompressible flow. What I am to emphasise is that the student has to understand how to simplify the problems, whether he has to solve the problem as a steady compressible flow or steady incompressible flow or you make it total unsteady compressible flow, which again you have to do volume integral to solve this.

The simplification matters a lot to solve the problem as compared to using the advanced mathematics, try to understand this.

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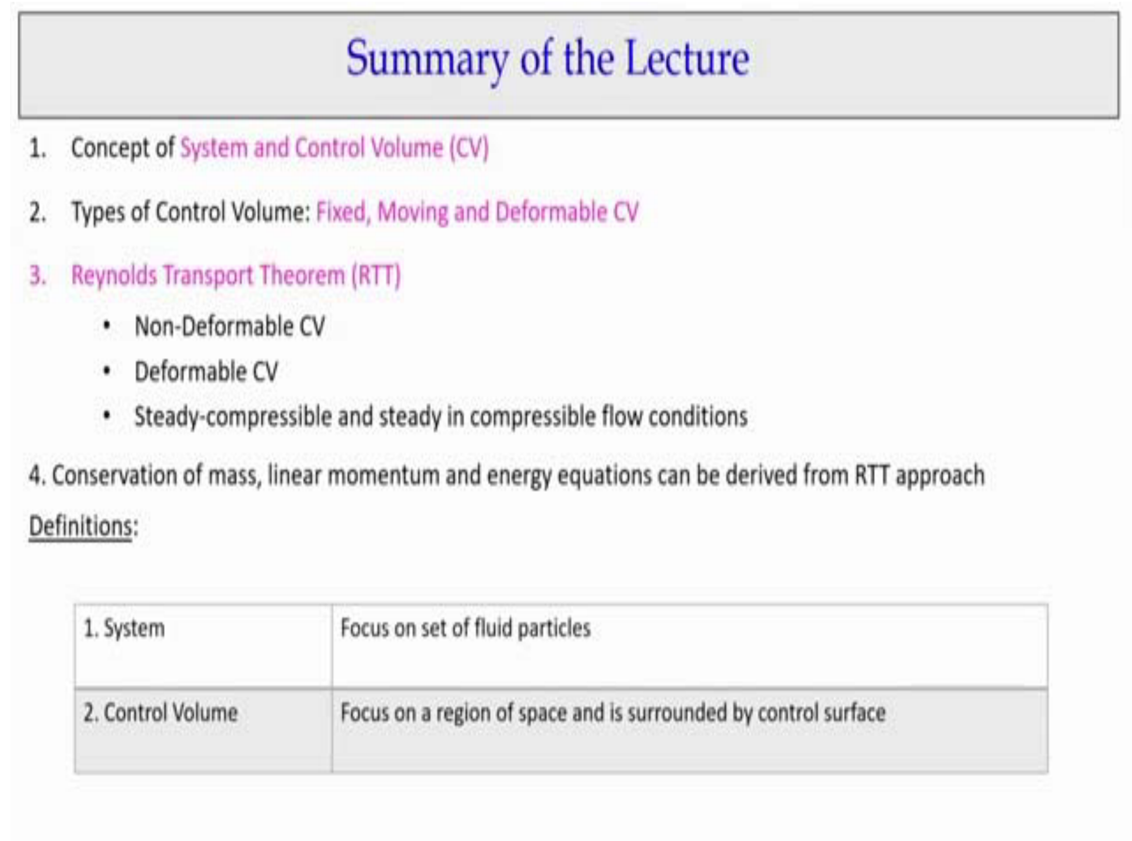

That is what I will try to give a lecture on that, how you develop an art to simplify a complex problem using the control volume concept. With these things let me summarise today's lecture. We discussed about system and control volume. We talked about fixed and deformable and moving control volume concept. The more important thing is that we derived the Reynolds transport theorem which can be used for fixed control volume, deformable control volume, moving control volume.

And we also demonstrated the use of the simplification of the steady problems. We simplified the problems as compared to go for unsteady incompressible or unsteady compressible flow which are complex problems where we need to integrate surface integration and volume integrations and solve this problem which is more complicated than these things.

So, we can have conservation of mass and linear momentum energy equations. We can derive it from RTT. That is what we will do in the next class, and as system wise I can again put this focus on the set of the fluid particles. We talked about the region of the space which is bounded or surrounded by the control surface. We will discuss in more detail about how RTT will be used to derive this mass linear momentum energy equations in the next class. Thank you a lot for this.